22 Dynamical SUSY Breaking: Part II

22.1 SUSY Breaking from Baryon Runaways

Consider a generalization of the 3-2 model:

	$Sp(2N)$	$SU(2N-1)$	$SU(2N-1)$	$U(1)$	$U(1)_R$
Q	$\square$	$\square$	$\mathbf{1}$	1	1
L	$\square$	$\mathbf{1}$	$\square$	-1	$-\frac{3}{2N-1}$
$\overline{U}$	$\mathbf{1}$	$\overline{\square}$	$\overline{\square}$	0	$\frac{2N+2}{2N-1}$
$\overline{D}$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$	-6	$-4N$

with a tree-level superpotential

$$W = \lambda Q L \overline{U}. \tag{22.1}$$

If we turn off the $SU(2N-1)$ gauge coupling and the superpotential, $Sp(2N)$ is in a non-Abelian Coulomb phase for $N \geq 6$, it has a weakly-coupled dual description for $N = 4, 5$, it s-confines for $N = 3$, and confines with a quantum-deformed moduli space for $N = 2$. If we turn off the $Sp(2N)$ gauge coupling and the superpotential, $SU(2N-1)$ s-confines for any $N \geq 2$. We will consider the case that $\Lambda_{SU} \gg \Lambda_{Sp}$.

Including the effects of the tree-level superpotential, this theory has a classical moduli space that can be parameterized by the gauge-invariants

	$SU(2N-1)$	$U(1)$	$U(1)_R$
$M = LL$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	-2	$-\frac{6}{2N-1}$
$B = \overline{U}^{2N-2}\overline{D}$	$\square$	-6	$-\frac{4(N^2-N+1)}{2N-1}$
$b = \overline{U}^{2N-1}$	$\mathbf{1}$	0	$2N+2$

subject to the constraints

$$M_{jk} B_l \epsilon^{klm_1 \cdots m_{2N-3}} = 0, \qquad M_{jk} b = 0. \tag{22.2}$$

These constraints split the moduli space into two branches: on one of them $M = 0$ and $B, b \neq 0$, and on the other $M \neq 0$ and $B, b = 0$.

First consider the branch where $M = 0$. In terms of the elementary fields, this corresponds to the VEVs (up to gauge and flavor transformations)

$$\langle \overline{U} \rangle = \begin{pmatrix} v\cos\theta & \\ & v\mathbf{1}_{2N-2} \end{pmatrix}, \qquad \langle \overline{D} \rangle = \begin{pmatrix} v\sin\theta \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \tag{22.3}$$

For $v \gg \Lambda_{SU}$, $SU(2N-1)$ is generically broken and the superpotential gives masses to Q and L or order λv. The low-energy effective theory is pure $Sp(2N)$, which has gaugino condensation.

$$\Lambda_{\text{eff}}^{3(2N+2)} = \Lambda_{Sp}^{3(2N+2)-2(2N-1)} \left(\lambda \overline{U}\right)^{2(2N-1)} \tag{22.4}$$

$$W_{\text{eff}} = \Lambda_{\text{eff}}^3 \sim \Lambda_{Sp}^3 \left(\frac{\lambda \overline{U}}{\Lambda_{Sp}}\right)^{\frac{2N-1}{N+1}} \tag{22.5}$$

For $N > 2$ this forces $\overline{U}$ to zero.

Now consider $v \ll \Lambda_{SU}$, then $SU(2N-1)$ s-confines so we have the following effective theory

	$Sp(2N)$	$SU(2N-1)$
L	$\square$	$\square$
$(Q\overline{U})$	$\square$	$\overline{\square}$
$(Q\overline{D})$	$\square$	$\mathbf{1}$
(Q^{2N-1})	$\square$	$\mathbf{1}$
B	$\mathbf{1}$	$\square$
b	$\mathbf{1}$	$\mathbf{1}$

with

$$W = \lambda(Q\overline{U})L + \frac{1}{\Lambda_{SU}^{4N-3}} \left[(Q^{2N-1})(Q\overline{U})B + (Q^{2N-1})(Q\overline{D})b - \det \overline{Q}Q\right] \tag{22.6}$$

So $(Q\overline{U})$ and L can be integrated out with $(Q\overline{U}) = 0$, and we have

$$W = \frac{1}{\Lambda_{SU}^{4N-3}} (Q^{2N-1})(Q\overline{D})b \tag{22.7}$$

On this branch of the moduli space $\langle b \rangle \sim \langle \overline{U}^{2N-1} \rangle \neq 0$, and this VEV gives a mass to (Q^{2N-1}) and $(Q\overline{D})$ which leaves an pure $Sp(2N)$ as the low energy effective theory.

$$\Lambda_{\text{eff}}^{3(2N+2)} = \Lambda_{Sp}^{3(2N+2)-2(2N-1)} \left(\lambda \Lambda_{SU}\right)^{2(2N-1)} \left(\frac{b}{\Lambda_{SU}}\right)^2 \tag{22.8}$$

$$W_{\text{eff}} = \Lambda_{\text{eff}}^3 \sim b^{\frac{1}{N+1}} \left(\Lambda_{Sp}^{N+4} \lambda^{2N-1} \Lambda_{SU}^{(2N-2)}\right)^{\frac{1}{N+1}} \tag{22.9}$$

Which forces $b \to \infty$, but the effective theory is only valid up to Λ_{SU} and we have seen that beyond this point the potential starts to rise, so the vacuum is around

$$\langle \overline{U}^{2N-1} \rangle \sim \Lambda_{SU}^{2N-1} \tag{22.10}$$

With some more work one can also see that SUSY is broken when $\Lambda_{Sp} \gg \Lambda_{SU}$. An especially interesting case is when $N = 3$, and $Sp(6)$ s-confines and we have the following effective theory

	$SU(5)$	$SU(5)$
(QQ)	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	$\mathbf{1}$
(LL)	$\mathbf{1}$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$
(QL)	$\Box$	$\Box$
(Q^{2N-1})	$\Box$	$\mathbf{1}$
$\overline{U}$	$\overline{\Box}$	$\overline{\Box}$
$\overline{D}$	$\overline{\Box}$	$\mathbf{1}$

with

$$W = \lambda(QL)\overline{U} + Q^{2N-1}L^{2N-1} \tag{22.11}$$

After integrating out (QL) and $\overline{U}$ we find $SU(5)$ with and antisymmetric tensor and an antifundamental and some singlets, which we have already seen breaks SUSY by other methods.

On the other branch where $\langle LL\rangle \neq 0$, one can see that the D-flat directions for L break $Sp(2N)$ to $SU(2)$, the effective theory is

	$SU(2)$	$SU(2N-1)$
Q'	$\Box$	$\Box$
L'	$\Box$	$\mathbf{1}$
$\overline{U}'$	$\mathbf{1}$	$\overline{\Box}$
$\overline{D}$	$\mathbf{1}$	$\overline{\Box}$

and some singlets with

$$W = \lambda Q'\overline{U}'L' \tag{22.12}$$

This is a generalized 3-2 model with a dynamical superpotential, for $\langle L\rangle \gg \Lambda_{SU}$ the vacuum energy is independent of the $SU(2)$ scale and proportional to $\Lambda^4_{SU,\mathrm{eff}} \propto$ a positive power of $\langle L\rangle$, which drives $\langle L\rangle$ smaller. For $\langle L\rangle \ll \Lambda_{SU}$ we can use the s-confined description above, and find that the baryon b runs away. For $\langle L\rangle \approx \Lambda_{SU}$, the vacuum energy is

$$V \sim \Lambda^4_{SU} \tag{22.13}$$

which is larger than the vacuum energy on the other branch, so we see that the baryon runaway solution is the global minimum.

22.2 SUSY Breaking and Deformed Moduli Spaces

The Intriligator-Thomas-Izawa-Yanagida model is a vector-like theory which consists of the following fields

	$SU(2)$	$SU(4)$
Q	$\square$	$\square$
S	$\mathbf{1}$	$\begin{array}{c}\square\\[-2pt]\square\end{array}$

with

$$W = \lambda S^{ij} Q_i Q_j \tag{22.14}$$

The strong $SU(2)$ dynamics enforces a constraint

$$\mathrm{Pf}(QQ) = \Lambda^4 \tag{22.15}$$

while the equation of motion for S is

$$\frac{\delta W}{\delta S^{ij}} = \lambda Q_i Q_j = 0 \tag{22.16}$$

Since these equations are incompatable we see that SUSY is broken. Another way to see this is that for large values of λS we can integrate out the quarks and get gaugino condensation:

$$\begin{aligned}
\Lambda_{\text{eff}}^{3N} &= \Lambda^{3N-2}\,(\lambda S)^2 &(22.17)\\
W_{\text{eff}} &= \Lambda_{\text{eff}}^{3} = \Lambda^2 \lambda S &(22.18)\\
\frac{\delta W_{\text{eff}}}{\delta S^{ij}} &= \lambda \Lambda^2 &(22.19)
\end{aligned}$$

For general values of λS we can write:

$$W_{\text{eff}} = \lambda S^{ij} Q_i Q_j + X(\mathrm{Pf} M - \Lambda^4) \tag{22.20}$$

The potential is

$$V = \sum_i \Big|\frac{\delta W_{\text{eff}}}{\delta Q_i}\Big|^2 + \sum_{ij} \Big|\frac{\delta W_{\text{eff}}}{\delta S^{ij}}\Big|^2 \ . \tag{22.21}$$

For $\lambda \ll 1$ we have essentially solved this theory since we can take λS as a mass term, then

$$M_{ij} = \left(\mathrm{Pf}(\lambda S)\Lambda^{3N-F}\right)^{\frac{1}{N}} \left(\frac{1}{\lambda S}\right)_{ij} \tag{22.22}$$

$$
\begin{aligned}
V &= \sum_{ij} |\frac{\delta W_{\rm eff}}{\delta S^{ij}}|^2 = |\lambda|^2 \sum_{ij} |M_{ij}|^2 \\
&= |\lambda|^2 |{\rm Pf} S \Lambda^4| \sum_{ij} | \left(\frac{1}{S}\right)_{ij} |^2 \ ,
\end{aligned}
\tag{22.23}
$$

which is minimized at

$$
S^{ij} = ({\rm Pf} S)^{\frac{1}{2}} \delta^{ij}
\tag{22.24}
$$

so

$$
V = |\lambda|^2 \Lambda^4
\tag{22.25}
$$

which agrees with our gaugino condensation calculation, so S appears to be a flat direction. This is Witten's loop-hole in the index argument since the theory with $\Delta W = m_s S^2$ is different from the theory with $m_s \to 0$ since vacua can come in from ∞.

As we saw in the O'Raifeartaigh model flat directions become pseudo-flat with SUSY breaking. For large values of λS in this model there is a wavefunction renormalization

$$
Z = 1 + c\lambda\lambda^\dagger \ln\left(\frac{\mu_0^2}{\lambda^2 S^2}\right)
\tag{22.26}
$$

So the vacuum energy is corrected to be

$$
\begin{aligned}
V &= \frac{|\lambda|^2}{|Z| \Lambda^4} \\
&\approx |\lambda|^2 \Lambda^4 \left[1 + c\lambda\lambda^\dagger \ln\left(\frac{\lambda^2 S^2}{\mu_0^2}\right)\right]
\end{aligned}
\tag{22.27}
$$

So the potential slopes towards the origin. This can be stabilized by gauging a subgroup of $SU(4)$. Otherwise there is a calculable O'Raifeartaigh model near $\lambda S \approx 0$, which breaks down near $\lambda S \approx \Lambda$. The behavior near this region is unknown.

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